HyperQube: A QBF-Based Bounded Model Checker for Hyperproperties

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Abstract. Hyperproperties are properties of systems that relate multiple computation traces, including many important security and concurrency properties. In this paper, we present HyperQube, a fully automated QBF-based bounded model checker for hyperproperties. HyperQube supports one-click system verification of NuSMV models with hyperproperties specified as HyperLTL logic formulas. The QBF-based technique allows HyperQube to deal with quantifier alternation. Based on the selection of either bug hunting or find witness, the instances of counterexamples or witnesses are also provided back to the user. We report on successful and efficient model checking using HyperQube, with a rich set of experiments on a variety of case studies, including security, concurrent data structures, path planning for robots, and testing.

1 Introduction

Hyperproperties \cite{11} have been shown to be a powerful framework for specifying and reasoning about important classes of requirements that were not possible with trace-based languages such as the classic temporal logics. Examples include information-flow security, consistency models in concurrent computing \cite{5}, and robustness models in cyber-physical systems \cite{6,33}. The temporal logic HyperLTL \cite{10} extends LTL by allowing explicit and simultaneous quantification over execution traces, describing the property of multiple traces. For example, the security policy observational determinism can be specified by the following HyperLTL formula: $\forall \pi_A, \forall \pi_B. (o_{\pi_A} \leftrightarrow o_{\pi_B}) \mathcal{W} \neg(i_{\pi_A} \leftrightarrow i_{\pi_B})$ which stipulates that every pair of traces $\pi_A$ and $\pi_B$ have to agree on the value of the (public) output $o$ as long as they agree on the value of the (secret) input $i$, where \(\mathcal{W}\) denotes the weak until operator.

There has been a recent surge of model checking techniques for HyperLTL specifications \cite{10,13,23,25}. These approaches employ various techniques (e.g., alternating automata, model counting, strategy synthesis, etc) to verify hyperproperties. However, they generally fall short in proposing a general push-button method to deal with identifying bugs with respect to HyperLTL formulas involving quantifier alternation. Indeed, quantifier alternation has been shown to generally elevate the complexity class of model checking HyperLTL specifications in different shapes of models \cite{2,10}.
In this paper we present the tool HyperQube, a fully automated bounded model checker for hyperproperties. In general, HyperQube works as follows: (1) parse the model(s) into Boolean encoding, (2) unroll the encoding together with a translated HyperLTL formula up to a certain bound, and (3) solve the unrolling using a QBF solver, and gives the BMC result according to the satisfiability of the unrolling formulas.

**Architecture of the Tool.** The architecture overview of HyperQube is presented in Fig. 1. The first layer contains two inputs from the user: a (set of) NuSMV model(s), and a hyperproperty defined in our grammar that the user want to do verification or synthesis with. The second layer contains a parser for the models and a translator for the hyperproperty, which convert each part into Boolean formulas representations. The third layer collects the Boolean encodings, unroll the encodings according to the user defined bounded semantics, and add quantifications over variables to make it quantified Boolean formula (QBF). Finally, the fourth layer is a QBF solver QuAbS, which solves the unrolled QBF formulas. The solver will either output the satisfaction or dissatisfaction result, and with counterexample(s) if needed based on the quantifier selection.

The current version of HyperQube does not incorporate a loop condition, as implementing such a condition for multiple traces is not straightforward. This, of course, comes at the cost of lack of a completeness result.

While our QBF encoding is a natural generalization of BMC for HyperLTL, the main contribution of this paper is to present a tool with a more refined view of how to interpret the behavior of the formula beyond the unrolling depth $k$. In HyperQube, we use a method that allows to interpret a wide range of outcomes of the QBF solver and relate these to the original model checking decision problem. To this end, we present the following semantics for BMC for HyperLTL:
• *Pessimistic* semantics (like in LTL BMC) under which pending eventualities are considered to be unfulfilled. This semantics works for *sometimes finitely satisifiable* (SFS) temporal formulas and paves the way for bug hunting.

• *Optimistic* semantics considers the dual case, where pending eventualities are assumed to be fulfilled at the end of the trace. This semantics works for *sometimes finitely refutable* (SFR) formulas, and allows us to interpret unsatisfiability of QBF as proof of correctness even with bounded traces.

• *Halting* variants of the optimistic and pessimistic semantics, which allow sound and complete decision on a verdict for terminating models.

**Running Example.** We now present a simple example to show the applicability of HyperQube in real-world program verification. Consider the program presented in Algorithm 1. The inputs are two boolean variables, high and low, representing high-security and low-security variables respectively. The value high is nondeterministically set to either true or false initially, and the value of low was set to false first then change according to the if-else clauses. This program can be represented as a Kripke structure as Fig. 2. Next, consider the security policy *non-interference* can be specified by the following HyperLTL formula:

$$
\varphi_{\text{NI}} = \forall \pi_A. \exists \pi_B. \bigcirc (\text{high}_{\pi_A} \not\leftrightarrow \text{high}_{\pi_B}) \land \Box (\text{low}_{\pi_A} \leftrightarrow \text{low}_{\pi_B})
$$

It stipulates that for all traces $\pi_A$, there must exists a $\pi_B$ such that their high-secure value high are different, but the low-security observation low always stay the same. Satisfying $\varphi_{\text{NI}}$ infers that one cannot guess the secret value by observing the public information.

In this example, it is easy to observe that in Algorithm 1, since the value of low variable is depending on the value of high variable; hence, a malicious attacker can obtain the actual value of high variable by observing the low variable. That is, the program violates non-interference. For example, the path $s_0, s_1, s_2^*$ is a counterexample that violates the hyperproperty $\varphi_{\text{NI}}$.

HyperQube serves as a one-click tool to verify/falsify a variety of hyperproperties with given model(s) as presented in our running example. Given a model with a hyperproperty, one can check whether the model satisfies or violate the given specification. Besides model checking verdict, HyperQube also returns either witness(es) to the verified cases, or counterexample(s) to the falsified cases.
Contributions. In summary, the contributions of this paper are:

1. **The HyperQube Bounded Model Checker for Hyperproperties**: We present the fully implemented tool HyperQube that is able to do bounded model checking for general hyperproperties. The main advantage of our approach is by leverage on the technique of QBF-solving, HyperQube is capable of handling HyperLTL formulas with quantifier alternation, which is usually a source of difficulty in model checking hyperproperties.

2. A rich set of case studies of HyperQube. Our experimental evaluation includes a rich set of case studies, such as information-flow security, linearizability in concurrent data structures, path planning in robotic applications, and mutation testing. Our evaluation shows that our technique is effective and efficient in identifying bugs in several prominent examples. We also show that our QBF-based approach is certainly more efficient than a brute-force SAT-based approach, where universal and existential quantifiers are eliminated by combinatorial expansion to conjunctions and disjunctions. We also show that in some cases our approach can also be used as a tool for synthesis. Indeed, a witness to an existential quantifier in a HyperLTL formula is an execution path that satisfies the formula. For example, our experiments on path planning for robots showcase this feature of HyperQube.

The rest of the paper is structured as follows. Section 2 contains the overview of our tool HyperQube. Section 3 introduces the tool design and implementation in details. Section 3 presents an empirical evaluation of our tool HyperQube. Section 5 presents the related work and Section 6 concludes.

## 2 Overview of HyperQube Algorithm

In this section, we present the core algorithm used in the tool HyperQube. As the data-flow presented in Fig. 1, the logic of HyperQube works as follows: Given a HyperLTL formula, e.g., $\varphi = \forall \pi_A.\exists \pi_B.\psi$, and a family of Kripke structures $K = (K_A, K_B)$ (one per trace variable), HyperQube performs in the following steps. First, the transition relation of $K_\pi$ (for every $\pi$) is parsed into a Boolean encoding $[[K_\pi]]$ using a parser written in Python using a Python library pyNuSMV. Secondly, the inner LTL subformula $\psi$ is also parsed into Boolean representation $[[\psi]]$ in a similar fashion to the BMC unrolling technique for LTL using a function genqbf written in Ocaml. This way, the QBF encoding for a bound $k \geq 0$ roughly appears as:

$$[[K, \neg \varphi]]_k = \exists \overline{x_A}.\forall \overline{x_B}.[[K_A]]_k \land ([K_B]_k \rightarrow [[\neg \psi]]_k),$$

where the vector of Boolean variables $\overline{x_A}$ (respectively, $\overline{x_B}$) are used to represent the states and propositions of $K_A$ (resp. $K_B$) for steps from 0 to $k$. Formulas $[[K_A]]_k$ and $[[K_B]]_k$ are the unrollings $K_A$ (using $\overline{x_A}$) and $K_B$ (using $\overline{x_B}$), and $[[\neg \psi]]$ (that uses both $\overline{x_A}$ and $\overline{x_B}$) is the fixpoint Boolean encoding of $\neg \psi$. In the following subsections, we describe in detail (1) the bounded semantics, and (2) how to generate a QBF query from an instance of the model checking problem.
### 2.1 Bounded Semantics for BMC

We introduce now the bounded semantics of HyperLTL, used in Subsection 2.2 to generate queries to a QBF solver to aid solving the model checking problem.

<table>
<thead>
<tr>
<th>semantics</th>
<th>intuition</th>
<th>infinite inference</th>
</tr>
</thead>
<tbody>
<tr>
<td>pessimistic</td>
<td>A formula is declared false unless it is witnessed to be true within the bound explored.</td>
<td>If $(T, H, 0) \models^p_{k} \varphi$, then $(T, H, 0) \models \varphi$</td>
</tr>
<tr>
<td>optimistic</td>
<td>A formula is declared true unless it is witnessed to be false within the bound explored.</td>
<td>If $(T, H, 0) \not\models^{opt}_{k} \varphi$, then $(T, H, 0) \not\models \varphi$</td>
</tr>
<tr>
<td>halting pessimistic</td>
<td>A formula is declared false unless it is witnessed to be true before the program halts.</td>
<td>If $(T, H, 0) \models^{hp}_{k} \varphi$, then $(T, H, 0) \models \varphi$</td>
</tr>
<tr>
<td>halting optimistic</td>
<td>A formula is declared true unless it is witnessed to be false before the program halts.</td>
<td>If $(T, H, 0) \not\models^{hp}_{k} \varphi$, then $(T, H, 0) \not\models \varphi$</td>
</tr>
</tbody>
</table>

**Table 1:** Four Different Bounded Semantics.

**Bounded Semantics.** We assume the HyperLTL formula is closed and of the form $Q_A \pi_A, Q_B \pi_B \ldots Q_Z \pi_Z, \psi$, where $Q \in \{\forall, \exists\}$ and it has been converted into negation-normal form (NNF) so that the negation symbol only appears in front of atomic propositions, e.g., $\neg a_{\pi_A}$. Without loss of generality and for the sake of clarity from other numerical indices, we use roman alphabet as indices of trace variables. Thus, we assume that $\text{Vars}(\varphi) \subseteq \{\pi_A, \pi_B, \ldots, \pi_Z\}$. The main idea of BMC is to perform incremental exploration of the state space of the systems by unrolling the systems and the formula up-to a bound.

Let $k \geq 0$ be the unrolling bound and let $T = \langle T_A \ldots T_Z \rangle$ be a tuple of sets of traces, one per trace variable. We start by defining a satisfaction relation between HyperLTL formulas for a bounded exploration $k$ and models $(T, H, i)$, where $T$ is the tuple of set of traces, $H$ is a trace assignment mapping, and $i \in \mathbb{Z}_{\geq 0}$ that points to the position of traces. We define four different bounded semantics as presented in Fig. 1 Intuitively, bounded semantics are different strategies to predict the unseen future with finite observations. All these semantics coincide in the interpretation of quantifiers, Boolean connectives, and temporal operators up-to instant $k - 1$, but differ in their assumptions about unseen future events after the bound of observation $k$.

Observe that the pessimistic semantics is the semantics in the traditional BMC for LTL. In the pessimistic semantics a formula is declared false unless it is witnessed to be true within the bound explored. In other words, formulas can only get “truer” with more information obtained by a longer unrolling. Dually, the optimistic semantics considers a formula true unless there is evidence within the bounded exploration on the contrary. Therefore, formulas only get “falsers” with further unrolling. For example, formula $\Box p$ always evaluates to false in
the pessimistic semantics. In the optimistic semantics, it evaluates to true up-to bound \( k \) if \( p \) holds in all states of the trace up-to and including \( k \). However, if the formula evaluates to false at some point before \( k \), then it evaluates to false for all \( j \geq k \). The following lemma formalizes this intuition in HyperLTL.

In turn, the verdict obtained from the exploration up-to \( k \) can (in some cases) be used to infer the verdict of the model checking problem. As in classical BMC, if the pessimistic semantics find a model, then it is indeed a model. Dually, if our optimistic semantics fail to find a model, then there is no model. The next lemma formally captures this intuition.

### 2.2 QBF Encoding

Given a family of Kripke structures \( \mathcal{K} \), a HyperLTL formula \( \varphi \), and bound \( k \geq 0 \), our goal is to construct a QBF formula \([\mathcal{K}, \varphi]_k\) whose satisfiability can be used to infer whether or not \( \mathcal{K} \models \varphi \). Due to limited space, in the following paragraphs, we use our running example to describe how to encode the model and the formula, and then how to combine the two to generate the final QBF query.

**Encoding the models.** First, note that the state space \( S \) can be encoded with a (logarithmic) number of bits in \(|S|\). We introduce additional variables \( n_0, n_1, \ldots \) to encode the state of the Kripke structure and use \( \text{AP}^* = \text{AP} \cup \{n_0, n_1, \ldots\} \) for the extended alphabet that includes the encoding of \( S \). In this manner, the set of initial states of a Kripke structure is a Boolean formula over \( \text{AP}^* \). Consider the program introduced in Algorithm 1. The initial condition \( I_A \) can be encoded as follows:

\[
I_A := (\neg n_0 \land \neg n_1 \land \neg n_2) \land (\text{high} \lor \neg \text{high}) \land \neg \text{low} \land \neg \text{halt}
\]

That is, \((\neg n_0 \land \neg n_1 \land \neg n_2)\) represents state \( s_0 \), the value of \( \text{high} \) can be initially set to either true or false, \( \text{low} \) is false, and the program does not halt yet. All other states can be encoded in the same manner.

Similarly, \( R_A \) is a binary relation that encodes the transition relation \( \delta \) of \( K_A \). The encoding into QBF works by introducing fresh Boolean variables (a new copy of \( \text{AP}^* \) for each Kripke structure \( K_A \) and position), and then producing a Boolean formula that encodes the unrolling up-to \( k \). We use \( x_A^i \) for the set of fresh copies of the variables \( \text{AP}^* \) of \( K_A \) corresponding to position \( i \in [0, k] \).

We use \( I_A(x) \) for the Boolean formula (using variables from \( x \)) that encodes the initial states, and \( R_A(x, x') \) (for two copies of the variables \( x \) and \( x' \)) for the Boolean formula whether \( x' \) encodes a successor states of \( x \). For example, for \( k = 3 \), we unroll the transition relation up-to 3 as follows,

\[
[K_A]_3 = I_A(x_A^0) \land R_A(x_A^0, x_A^1) \land R_A(x_A^1, x_A^2) \land R_A(x_A^2, x_A^3)
\]

which is the Boolean formula representing valid traces of length 4, using four copies of the variables \( \text{AP}^*_A \) that represent the Kripke structure \( K_A \).
Encoding the inner LTL formula. The idea of the construction of the inner LTL formula is analogous to standard BMC as well, except for the choice of different semantics described in Subsection 2.1. Consider the non-interference formula $\phi_{NI}$ we introduced in Sec. 1. With the negated formula with bound $k = 3$, we have:

\[
\llbracket (\bigcirc (\text{high}_{\pi_A} \leftrightarrow \text{high}_{\pi_B}) \lor \bigdiamond (\text{low}_{\pi_A} \not\leftrightarrow \text{low}_{\pi_B})) \rrbracket :=
(\text{high}^1_{\pi_A} \leftrightarrow \text{high}^1_{\pi_B}) \lor ((\text{low}^0_{\pi_A} \not\leftrightarrow \text{low}^0_{\pi_B}) \lor 
(\text{low}^1_{\pi_A} \not\leftrightarrow \text{low}^1_{\pi_B}) \lor (\text{low}^2_{\pi_A} \not\leftrightarrow \text{low}^2_{\pi_B}) \lor (\text{low}^3_{\pi_A} \not\leftrightarrow \text{low}^3_{\pi_B}))
\]

Complete Formula. Finally, to combine the model description with the encoding of the HyperLTL formula, we use two identical copies of the given Kripke structure to represent different paths $\pi_A$ and $\pi_B$ on the model, denoted as $K_A$ and $K_B$. The final resulting formula is:

\[
\llbracket K, \neg \phi_{NI} \rrbracket_k := \exists \forall x_A. \forall x_B. (\llbracket K_A \rrbracket_k \land (\llbracket K_B \rrbracket_k \rightarrow [\neg \phi_{NI}]_{pes}^{0,k}))
\]

The satisfaction result shows that $\llbracket K, \neg \phi_{NI} \rrbracket_k^{pes}$ is true, indicating that a witness of violation is found. According to the pessimistic semantics, a successful detection of a counterexample infers that the formula is also true in infinity; that is, $K \models_k \neg \phi$ implies $K \models \neg \phi$ in infinite semantics. From here, it allows to conclude that $K \models \neg \phi$.

3 The HyperQube Tool Implementation

In this section, we explain the design and implementation of HyperQube. As presented in Fig. 1, we first translate the user inputs of model(s) and specification into boolean formulas, unroll the formulas based on the selected bounded semantics, and check the satisfiability using existing QBF solver. In the following subsections, we elaborate each step in details.

3.1 Model Description

HyperQube reads input model descriptions which are written in the language of NuSMV [9]. NuSMV is a symbolic model checker which supports standard LTL and CTL model checking, but does not accept hyperproperties such as HyperLTL formulas. However, we notice that since the internal mechanism of NuSMV is based on Binary Decision Diagrams (BDDs), the way NuSMV store model information (i.e., initial condition, transition relations, etc.) are similar to our target QBF encoding as illustrated in Section 2.2. As a result, by leveraging on the BDDs encoding feature, the idea is to extract the explicit state information by accessing the internal data structure of NuSMV, then encode the state transitions into our QBF formulations. To achieve this, we utilize an existing python library
pyNuSMV [8] that provides model initialization and method access of NuSMV. We next explain how we use pyNuSMV to convert a NuSMV model to Boolean formulas.

Our code is implemented in parser.py. We first load the given NuSMV model using the `complute_model()` function in pyNuSMV library to create an instance of the given model. Next, our parser will analyze the type of each defined variable to help decide whether we need to bit-blast the variable or not. That is, for each numerical variable $v_i$, the parser will automatically generate $\log(\max(v_i))$ number of Boolean variables in order to bit-blast the variable, where $\max(v_i)$ is a function that returns the maximum value of $v_i$ which is defined in the given NuSMV file. For example, define a variable called “state”, that ranges from 0 to 5, our parser will parse it to three boolean variables $state_2, state_1$, and $state_0$. In this way, when state is set to 0, 1, 2, 3 ..., the Boolean formulas can be expressed as $(\neg state_2 \land \neg state_1 \land \neg state_0)$, $(\neg state_2 \land \neg state_1 \land state_0)$, $(\neg state_2 \land state_1 \land \neg state_0)$, $(\neg state_2 \land state_1 \land state_0)$, ..., respectively. All other numerical variables defined in the given model can be similarly encoded.

The main purpose of using pyNuSMV package to parse NuSMV model into Boolean encoding is to provide the tool user a simple and convenient way to give model descriptions. Instead of directly asking the user to compose the whole boolean formulas such complicated logic as shown in Sec 3.1, the user can build the model in well-defined NuSMV language. Furthermore, although HyperQube is a QBF-based model checker, allowing NuSMV models as user input can reduce the redundant work required by the user (i.e., bit-blasting each numerical variable in the model). Another benefit of this approach is that HyperQube inherits the bounded definition of integers defined in NuSMV language. That is, the domain of each numerical variable appears in the HyperLTL formula, will always be bounded as well.

### 3.2 HyperLTL Specification

The specification of HyperQube follows the standard HyperLTL syntax. In order to incorporate our model encoding as illustrated in Subsection 3.1, we present a new grammar for the hyperproperty input of HyperQube, as presented in Fig. 3. Our grammar supports HyperLTL logic by specifying each trace using tid to their corresponding path variable, where each path variable can be universally or existentially quantified.

Next, for the inner LTL formula (i.e., the quantifier free segment), we support three different kinds of operations: arithmetic operations (`arith_op`), binary operations (`binary_op`), and unary prefix operations (`unary_prefix_op`), such that:

- arithmetic operations must be applied two arithmetic values;
- binary operations can only be applied on two boolean values; and,
- unary prefix operations must be followed by a single boolean value.
Fig. 3: Grammar for HyperLTL specification of HyperQube

For example, assume $PC$ is an integer variable that ranges from 0 to 10, representing the program counter of a thread, and $halt$ is a boolean variable, indicating whether the thread has reach to the halting status or not. The following input correctly follows the HyperQube grammar for hyperproperties,

$$\forall A \exists B \ (PC[A] = PC[B]) \ U \ (halt[A] \land halt[B])$$

However, the inputs such as $(PC[A] \land halt[B])$, or $(PC = halt[B])$, are incorrect formula inputs and cannot be accepted and translated by HyperQube. We assume the all variables in the input formulas are defined in the input NuSMV model. Furthermore, all variables appear in the input formula must match the definition in the given model. For example, if the program counter variable $PC$ is defined to range from 0 to 10, then the expression $(PC[A] = 20)$, is incorrect.

### 3.3 Unrolling of Model and HyperLTL Formula

Once the model description and the property specification are both converted into boolean formulas, the next step is to perform bounded model checking by unrolling the Boolean formula. The big-picture of Boolean formula unrolling is the same as we presented in Subsection 2.2. That is, unroll the transition relation from model parsing, together with the semantics of linear temporal logic from formula translating.

Our unrolling mechanism is written in the genqbf using Ocaml. It takes the Boolean expressions of model description and HyperLTL specification, as illustrated in Subsections 3.1 and 3.2. Then, genqbf will create multiple copies to build the complete formula as we demonstrated in Subsection 2.2. We also allow
genqbf to take care of multiple user-specified features, including the bound of unrolling, the selected bounded semantics, and the decision of whether the input HyperLTL formula should be negated (i.e., to perform counterexample hunting), or stays as it is (i.e., perform witness searching).

3.4 How to Use the Tool

**Tool Inputs.** The input model(s) must be written in correct NuSMV syntax. Any syntax error of the input model will be caught when HyperQube parses the model. The input HyperLTL formula should follow the correct grammar presented in 3 as the input specification. Any formula that are ‘incorrect’ (as we demonstrated in Subsection 3.2), will cause errors reported during executions. With the required inputs ready, the next step is to specify different user-defined flags for HyperQube, including:

- a natural number \( n \) specify the exploration bound of model checking, using the flag \(-k \ n\).
- selection of quantifiers that the current version of HyperQube support, using the flag \(-QS \{EE, AA, EA, AE\}\).
- choice of one of the bounded semantics as we introduced in Subsection 2.1, using the flag \(-sem \{pes, opt, h-pes, h-opt\}\).
- choice between \-bughunt or \-find to perform bug hunting or witness searching.

**Tool Outputs.** After executing HyperQube, the solver will return one of the cases as demonstrated in Tab. 2. In general, the flag \-bughunt and \-find are dual with each other, since one negate the formula and the other one does not. If counterexample(s) or witness(es) has been found, HyperQube will return the path with the value of each variable specified in each time step. For example, given a model \( K \) and a HyperLTL formula \( \varphi \) in the form of \( \forall \exists \) and perform bug hunting, a SAT result indicates that \( K \not\models \varphi \), with a counterexample which is a trace from \( k \) that violates \( \varphi \).

<table>
<thead>
<tr>
<th>HLTL</th>
<th>-bughunt (HLTL negated)</th>
<th>-find (original HLTL)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \exists \varphi )</td>
<td>( \varphi ) rejected. countere xamples.</td>
<td>( \varphi ) accepted, witnesses.</td>
</tr>
<tr>
<td>( \forall \varphi )</td>
<td>( \varphi ) refuted. countere xamples.</td>
<td>( \varphi ) accepted.</td>
</tr>
<tr>
<td>( \forall \exists \varphi )</td>
<td>( \varphi ) rejected. a countere xample.</td>
<td>( \varphi ) accepted verified.</td>
</tr>
<tr>
<td>( \exists \forall \varphi )</td>
<td>( \varphi ) rejected.</td>
<td>( \varphi ) accepted rejected, a countere xample.</td>
</tr>
</tbody>
</table>

**Table 2:** Different output interpretations of HyperQube for model checking \( K \models \varphi \).
4 Experimental Results and Analysis

We now evaluate HyperQube by a rich set of case studies on information-flow security, concurrent data structures, path planning for robots, and mutation testing. In this section, we will refer to each property in HyperLTL as in Table 3. We have implemented HyperQube using the technique described in Section 2: 1) Given a NuSMV model, the tool automatically parses the transition relation and unfolds it up to $k \geq 0$ by the home-grown procedure parser.py and genqbf. Given the choice of the semantics (pessimistic, optimistic, and halting variants) the unfolded transition relation is combined with the QBF encoding of the input HyperLTL formula to form a complete QBF instance which is then fed to the QBF solver QuAbS [28]. All experiments in this section are run on an iMac with Intel i7 CPU @3.4 GHz and 32 GB of RAM.

Case Study 1: Symmetry in Lamport’s Bakery algorithm [13]. Symmetry states that no specific process has special privileges in terms of a faster access to the critical section (see different symmetry formulas in table 3). In these for-
mula, each process $P_n$ has a program counter denoted by $pc(P_n)$, $select$ indicates which process is selected to process next, $pause$ if both processes are not selected, $sym\_break$ is which process is selected after a tie, and $sym(select_{\pi_A}, select_{\pi_B})$ indicates if two traces are selecting two opposite processes. The Bakery algorithm does not satisfy symmetry (i.e. $\varphi_{sym}$), because when two or more processes are trying to enter the critical section with the same ticket number, the algorithm always gives priority to the process with the smaller process ID. HyperQube returns SAT using the pessimistic semantics, indicating that there exists a counterexample in the form of a falsifying witness to $\pi_A$ in formula $\varphi_{sym}$, table 4 includes our result on other symmetry formulas presented in table 3.

The counterexample consists of a trace represents a witness trace of Bakery that violates An observable witness within finite bound is sufficient with the pessimistic semantics to infer that all future observations are consistently indicating the given model does not satisfy original property.

**Case Study 2: Linearizability in SNARK [15].** SNARK implements a concurrent double-ended queue using double-compare-and-swap (DCAS) and a doubly linked-list that stores values in each node. Linearizability [29] requires that any history of execution of a concurrent data structure (i.e., sequence of invocation and response by different threads) matches some sequential order of invocations and responses (see formula $\varphi_{lin}$ in table 3). SNARK is known to have two linearizability bugs and HyperQube returns SAT using the pessimistic semantics, identifying both bugs as two counterexamples. With the use of pessimistic semantics, a witness of linearizability violation of length $k$ is enough to infer that the given system does not satisfy the linearizability property. The bugs we identified are precisely the same as the ones reported in [15].

**Case Study 3: Non-interference in multi-threaded Programs.** Non-interference [26] states that low-security variables are independent from the high-security variables, thus preserving secure information flow. We consider the concurrent program example in [31], where $PIN$ is high security input and $Result$ is low security output. HyperQube returns SAT in the halting pessimistic semantics, indicating that there is a trace that we can detect the difference of a high-variable by observing a low variable, that is, violating non-interference. We also verified the correctness of a fix to this algorithm, proposed in [31] as well. HyperQube uses the UNSAT results from the solver (with halting optimistic semantics) to infer the absence of violation, that is, verification of non-interference.

**Case Study 4: Fairness in non-repudiation protocols.** A non-repudiation protocol ensures that a receiver obtains a receipt from the sender, called non-repudiation of origin ($NRO$), and the sender ends up having an evidence, named non-repudiation of receipt ($NRR$), through a trusted third party. A non-repudiation protocol is fair if both $NRR$ and $NRO$ are either received or not received by the parties (see formula $\varphi_{fair}$ in table 3). We verified two different protocols from [30], namely, $T_{incorrect}$ that chooses not to send out $NRR$ after receiving $NRO$, and a correct implementation $T_{correct}$ which is fair. For $T_{correct}$
(respectively, $T_{\text{incorrect}}$). HyperQube returns UNSAT in the halting optimistic semantics (respectively, SAT in the halting pessimistic semantics), which indicates that the protocol satisfies (respectively, violates) fairness.

**Case Study 5: Path planning for robots.** The use of HyperQube for robotic path planning is slightly different from the above-mentioned cases. We have used HyperQube beyond verification, to synthesize strategies for robotic planning [34]. Here, we focus on producing a strategy that satisfies two control requirements for a robot to reach a goal in a grid. First, the robot should take the shortest path (see formula $\varphi_{sp}$ in table 3). Fig. 4 shows a $10 \times 10$ grid, where the red, green, and black cells are initial, goal, and blocked cells, respectively. HyperQube returns SAT and the synthesized path is shown by the blue arrows.

We also used HyperQube to solve the path robustness problem, meaning that starting from an arbitrary initial state, a robot reaches the goal by following a single strategy (see formula $\varphi_{rb}$ in table 3). Again, HyperQube returns SAT for the grid shown in Fig. 5.

**Case Study 6: Mutation testing.** We adopted the model from [16] and apply the original formula that describes a good test mutant together with the model (see formula $\varphi_{mut}$ in table 3). HyperQube returns SAT, indicating successful finding of a qualified mutant. We note that in [16] the authors were not able to generate test cases via $\varphi_{mut}$, as the model checker MCHyper is not able to handle quantifier alternation in push-button fashion.

**Results and analysis.** Table 4 summarizes our results including running times, the bounded semantics applied, the output of the QBF solver, and the resulting infinite inference conclusion as presented in Table 1. As can be seen, our case studies range over model checking of different fragments of HyperLTL. It is important to note that HyperQube run time consists of generating a QBF formula by genqbf and then checking its satisfiability by QuAbS. It is remarkable that in some cases, QBF formula generation takes longer than checking its satisfiability. The models in our experiments also have different sizes. The most complex case study is arguably the SNARK algorithm, where we identify both bugs in the algorithm in 472 and 1497 seconds. In cases 5.1 – 6.2, we also demonstrate the ability of HyperQube to solve synthesis problems by leveraging the existential quantifier in a HyperLTL formula.

Finally, we elaborate more on scalability of the path planning problem for robots. This problem was first studied in [34], where the authors reduce the problem to SMT solving using Z3 [14] and by eliminating the trace quantifiers through a combinatorial enumeration of conjunctions and disjunctions. Table 5
compares our approach with the brute-force technique employed in [34] for different grid sizes. Our QBF-based approach clearly outperforms the solution in [34], in some cases by an order of magnitude.

<table>
<thead>
<tr>
<th>Model K</th>
<th>Formula bound k</th>
<th>AP⁺</th>
<th>QBF semantics</th>
<th>gensqbf [s]</th>
<th>QuAbS [s]</th>
<th>Total [s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bakery.3proc</td>
<td>ϕS₁</td>
<td>7</td>
<td>27</td>
<td>SAT</td>
<td>pes</td>
<td>0.44</td>
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<td>Bakery.3proc</td>
<td>ϕS₂</td>
<td>12</td>
<td>27</td>
<td>SAT</td>
<td>pes</td>
<td>1.31</td>
</tr>
<tr>
<td>Bakery.3proc</td>
<td>ϕS₃</td>
<td>20</td>
<td>27</td>
<td>UNSAT</td>
<td>opt</td>
<td>2.86</td>
</tr>
<tr>
<td>Bakery.3proc</td>
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<td>10</td>
<td>27</td>
<td>SAT</td>
<td>pes</td>
<td>0.86</td>
</tr>
<tr>
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<td>27</td>
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<td>pes</td>
<td>0.76</td>
</tr>
<tr>
<td>Bakery.5proc</td>
<td>ϕsym₁</td>
<td>10</td>
<td>45</td>
<td>SAT</td>
<td>pes</td>
<td>23.57</td>
</tr>
<tr>
<td>Bakery.5proc</td>
<td>ϕsym₂</td>
<td>10</td>
<td>45</td>
<td>SAT</td>
<td>pes</td>
<td>29.92</td>
</tr>
<tr>
<td>SNARK-bug1</td>
<td>ϕlin</td>
<td>26</td>
<td>160</td>
<td>SAT</td>
<td>pes</td>
<td>88.42</td>
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<tr>
<td>SNARK-bug2</td>
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<td>40</td>
<td>160</td>
<td>SAT</td>
<td>pes</td>
<td>718.09</td>
</tr>
<tr>
<td>3-Thread incorrect</td>
<td>ϕ₈ₘ</td>
<td>57</td>
<td>31</td>
<td>SAT</td>
<td>h-pes</td>
<td>19.56</td>
</tr>
<tr>
<td>3-Thread correct</td>
<td>ϕ₈ₘ</td>
<td>57</td>
<td>31</td>
<td>UNSAT</td>
<td>h-opt</td>
<td>23.91</td>
</tr>
<tr>
<td>NRP : T incorrect</td>
<td>ϕ₉₈</td>
<td>15</td>
<td>15</td>
<td>SAT</td>
<td>h-pes</td>
<td>0.10</td>
</tr>
<tr>
<td>NRP : T correct</td>
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<td>15</td>
<td>15</td>
<td>UNSAT</td>
<td>h-opt</td>
<td>0.08</td>
</tr>
<tr>
<td>Shortest Path</td>
<td>(see Table 5)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Initial State Robustness</td>
<td>Mutant</td>
<td>8</td>
<td>6</td>
<td>SAT</td>
<td>h-pes</td>
<td>1.40</td>
</tr>
</tbody>
</table>

Table 4: Performance of HyperQube, where column case# identifies the artifact, ✓ denotes satisfaction, and ✗ denotes violation of the formula. AP⁺ is the set of Boolean variables encoding K.

<table>
<thead>
<tr>
<th>Formula grid size</th>
<th>bound k</th>
<th>AP⁺</th>
<th>gensqbf [s]</th>
<th>QuAbS [s]</th>
<th>Total [s]</th>
<th>gensmt [s]</th>
<th>Z₃ [s]</th>
<th>Total[s]</th>
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<tr>
<td>ϕₛₚ</td>
<td>10³</td>
<td>20</td>
<td>12</td>
<td>1.30</td>
<td>0.57</td>
<td>1.87</td>
<td>8.31</td>
<td>0.33</td>
</tr>
<tr>
<td>20³</td>
<td>40</td>
<td>14</td>
<td>4.53</td>
<td>12.16</td>
<td>16.69</td>
<td>124.66</td>
<td>6.41</td>
<td>131.06</td>
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<tr>
<td>40³</td>
<td>80</td>
<td>16</td>
<td>36.04</td>
<td>35.75</td>
<td>71.79</td>
<td>1093.12</td>
<td>72.99</td>
<td>1166.11</td>
</tr>
<tr>
<td>60³</td>
<td>120</td>
<td>16</td>
<td>105.82</td>
<td>120.84</td>
<td>226.66</td>
<td>4360.75</td>
<td>532.11</td>
<td>4892.86</td>
</tr>
<tr>
<td>ϕᵣ₀</td>
<td>10³</td>
<td>20</td>
<td>12</td>
<td>1.40</td>
<td>0.35</td>
<td>1.75</td>
<td>11.14</td>
<td>0.45</td>
</tr>
<tr>
<td>20³</td>
<td>40</td>
<td>14</td>
<td>15.92</td>
<td>15.32</td>
<td>31.14</td>
<td>49.59</td>
<td>2.67</td>
<td>52.26</td>
</tr>
<tr>
<td>40³</td>
<td>80</td>
<td>16</td>
<td>63.16</td>
<td>20.13</td>
<td>83.29</td>
<td>216.16</td>
<td>19.81</td>
<td>235.97</td>
</tr>
</tbody>
</table>

Table 5: Path planning for robots and comparison to [34]. All cases use the halting pessimistic semantics and QBF solver returns SAT, meaning successful path synthesis.
5 Related Work

There has been a lot of recent progress in automatically verifying [13, 23–25] and monitoring [1, 5, 7, 21, 22, 27, 32] HyperLTL specifications. HyperLTL is also supported by a growing set of tools, including the model checker MChyper [13, 25], the satisfiability checkers EAHyper [20] and MGHyper [18], and the runtime monitoring tool RVHyper [21]. The complexity of model checking for HyperLTL for tree-shaped, acyclic, and general graphs was rigorously investigated in [2]. The first algorithms for model checking HyperLTL and HyperCTL* using alternating automata were introduced in [25]. These techniques, however, were not able to deal in practice with alternating HyperLTL formulas in a fully automated fashion. We also note that previous approaches that reduce model checking HyperLTL—typically of formulas without quantifier alternations—to model checking LTL can use BMC in the LTL model checking phase. However, this is a different approach than the one presented here, as these approaches simply instruct the model checker to use a BMC after the problem has been fully reduced to an LTL model checking problem while we avoid this translation. These algorithms were then extended to deal with hyperliveness and alternating formulas in [13] by finding a winning strategy in ∀∃ games. In this paper, we take an alternative approach by reducing the model checking problem to QBF solving, which is arguably more effective for finding bugs (in case a finite witness exists).

The satisfiability problem for HyperLTL is shown to be undecidable in general but decidable for the \( \exists^* \forall^* \) fragment and for any fragment that includes a \( \forall \exists \) quantifier alternation [17]. The hierarchy of hyperlogics beyond HyperLTL were studied in [12]. The synthesis problem for HyperLTL has been studied in [3] in the form of program repair, in [4] in the form of controller synthesis, and in [19] for the general case.

6 Conclusion and Future Work

We present HyperQube, a QBF-based bounded model checker for hyperproperties expressed in HyperLTL. We use the package pyNuSMV to parse the model into Boolean formulas, and unroll with the translated hyperproperties in our own defined grammar for HyperLTL. To this end, we proposed four different semantics that ensure the soundness of inferring the outcome of the model checking problem. To handle trace quantification in HyperLTL, we reduced the BMC problem to checking satisfiability of quantified Boolean formulas (QBF). This is analogous to the reduction of BMC for LTL to the simple propositional satisfiability problem. We have introduced different classes of semantics, beyond the pessimistic semantics common in LTL model checking, namely optimistic semantics that allow to infer full verification by observing only a finite prefix and halting variations of these semantics that additionally exploit the termination of the execution, when available. Through a rich set of case studies, we demonstrated the effectiveness and efficiency of HyperQube in verification of information-flow properties, linearizability in concurrent data structures, path planning in robotics, and fairness in non-repudiation protocols.
As for future work, our first step is to solve the loop condition problem. This is necessary to establish completeness conditions for BMC and can help cover even more examples efficiently. The application of QBF-based techniques in the framework of abstraction/refinement is another unexplored area. Success of BMC for hyperproperties inherently depends on effectiveness of QBF solvers. Even though QBF solving is not as mature as SAT/SMT solving techniques, recent breakthroughs on QBF have enabled the construction of HyperQube, and more progress in QBF solving will improve its efficiency.

References


